

Black Hole Relics in Large Extra Dimensions

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Recent calculations applying statistical mechanics indicate that in a setting with compactified large extra dimensions a black hole might evolve into a (quasi-)stable state with mass close to the new fundamental scale M_f . Black holes and therefore their relics might be produced at the LHC in the case of extra-dimensional topologies. In this energy regime, Hawking's evaporation scenario is modified due to energy conservation and quantum effects. We reanalyse the evaporation of small black holes including the quantisation of the emitted radiation due to the finite surface of the black hole. It is found that observable stable black hole relics with masses $\sim 1 - 3M_f$ would form which could be identified by a delayed single jet with a corresponding hard momentum kick to the relic and by ionisation, e.g. in a TPC.

The idea of Large eXtra Dimensions (LXDs) which was recently proposed in [1–5] might allow to study interactions at trans-planckian energies in the next generation collider experiments. Here, the hierarchy-problem is solved or at least reformulated in a geometric language by the existence of d compactified LXDs in which only the gravitons can propagate. The standard-model particles are bound to our 4-dimensional sub-manifold, often called our 3-brane.

The strength of a force at a distance r generated by a charge depends on the number of space-like dimensions. For distances smaller than the compactification length L , the gravitational interaction drops faster compared to the other interactions. For distances much bigger than L gravity is described by the well known potential law $\propto 1/r$. However, starting from $r \geq L$ the force lines are diluted into the extra dimensions resulting in a smaller effective coupling constant for gravity.

This scenario would lead to the following relation between the four-dimensional Planck mass m_p and the higher dimensional Planck mass M_f , which is the new fundamental scale of the theory

$$m_p^2 = L^d M_f^{d+2} \quad . \quad (1)$$

The lowered fundamental scale would induce a vast number of observable phenomena for quantum gravity at energies in the range M_f . In fact, the non-observation of these predicted features gives first constraints on the parameters of the model, the number of extra dimensions d and the fundamental scale M_f [6,7]. On the one hand, this scenario would have major consequences for cosmology and astrophysics such as the modification of inflation in the early universe and enhanced supernova-cooling due to graviton emission [3,8–11]. On the other hand, additional processes had to be expected in high-energy collisions [12]: production of real and virtual gravitons [13–17] and the creation of black holes at energies that can be achieved at colliders in the near future.

Especially the possibility of black hole production in LXDs at the LHC and from cosmic rays has received great attention [18–32]. Black holes produced in such interactions would be tiny and may decay on fm/c time scales [25]. Thus, the decay of these objects (black holes, p-branes, string balls) could be studied in detail in the laboratory. Unfortunately, very little is known about the final stages of black hole evaporation. Extensive speculations about the final fate of black holes have been brought forward in the literature and will be discussed in detail later. The general notion is that a small black hole stops evaporating particles when its mass approaches the Planck scale, resulting in the exciting possibility of forming a (quasi-)stable relic. In this letter, we study a model for the radiation from small black holes assuming a geometrical quantisation of the emitted radiation. The quantisation of the radiation can lead to black hole relics of masses around 1-3 TeV with small electric charge. Those relics might be observable at the LHC.

Let us start with the properties of black holes which may be accessible in the next generation colliders. A black hole which might be produced with $\sqrt{s} \approx 10$ TeV would have a radius much smaller than the size L of the LXDs [25]. In this case one can neglect the periodic boundary conditions due to compactification and approximate the space-time to be spherically symmetric. For these black holes, the metric is given by the $(d+4)$ -dimensional Schwarzschild metric [33].

Following Ref. [33] and implying the extra dimensions via Eq. (1), the Schwarzschild radius is given by

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$$R_S^{d+1} = \frac{2}{d+1} \frac{M}{M_f^{d+2}} \quad . \quad (2)$$

The metric takes the familiar form

$$ds^2 = -\gamma(r)dt^2 + \frac{1}{\gamma(r)}dr^2 + d\Omega_{(d+3)}^2 \quad (3)$$

with $d\Omega_{(d+3)}$ being the surface element of the $(d+3)$ -dimensional sphere, containing $(d+2)$ angles and $\gamma(r)$ given by

$$\gamma(r) = 1 - \left(\frac{R_S}{r} \right)^{d+1} \quad . \quad (4)$$

From this one gets the surface gravity:

$$\kappa = \frac{d+1}{2} \frac{1}{R_S} \quad , \quad (5)$$

which is the Newtonian force at the horizon in the Schwarzschild case. The surface of the black hole is

$$\mathcal{A} = \Omega_{(d+3)} R_S^{d+2} \quad , \quad (6)$$

with $\Omega_{(d+3)}$ denoting the surface of the unit $d+3$ -sphere

$$\Omega_{(d+3)} = \frac{2\pi^{\frac{d+3}{2}}}{\Gamma(\frac{d+3}{2})} \quad . \quad (7)$$

The production cross section for black holes in parton-parton or νN collisions can be estimated on geometrical grounds and is of order $\sigma(M) \approx \pi R_S^2$ [20,34–36]. Detailed studies to support this estimate can be found in the recent works by Jevicki and Thaler [37] and by Eardley and Giddings [38]. Assuming the validity of the classical approximation and setting $M_f \sim 1$ TeV and $d = 2$ one finds $\sigma \approx 1$ TeV⁻² ≈ 400 pb. The luminosity of pp interactions at the LHC would then allow the production of approximately $\approx 10^8$ black holes per year [22].

The fate of these small black holes is difficult to estimate. Their final evaporation is closely connected to the information loss puzzle. The black hole emits thermal radiation, whose sole property is the temperature, regardless of the initial state of the collapsing matter. So, if the black hole completely decays into statistically distributed particles, unitarity can be violated. This happens when the initial state is a pure quantum state and then evolves into a mixed state [39–41].

When one tries to avoid the information loss problem two possibilities are left. The information is regained by some unknown mechanism or a stable black hole remnant is formed which keeps the information. Besides the fact that it is unclear in which way the information should escape the horizon [42–47] there are several other arguments for black hole relics [48–51]:

- The uncertainty relation: The Schwarzschild radius of a black hole with Planck mass is of the order of the Planck length. Since the Planck length

is the wavelength corresponding to a particle of Planck mass, a problem arises when the mass of the black hole drops below Planck mass. Then one has trapped a higher mass, $M \geq M_f$, inside a volume which is smaller than allowed by the uncertainty principle [52]. To avoid this problem, Zel'dovich has proposed that black holes with masses below Planck mass should be associated with stable elementary particles [53]

- Corrections to the Lagrangian: The introduction of additional terms, which are quadratic in the curvature, yields a dropping of the evaporation temperature towards zero [75,55]. This holds also for extra dimensional scenarios [56] and is supported by calculations in the low energy limit of string theory [57,58].
- Further reasons for the existence of relics have been suggested to be black holes with axionic charge [59], the modification of the Hawking temperature due to quantum hair [60] or magnetic monopoles [61]. Coupling of a dilaton field to gravity also yields relics, with detailed features depending on the dimension of space-time [62,63].

Let us now compare the classical micro-canonical emission scenario to an approach that takes into account the effects of the geometrical quantisation of the emitted radiation. Note that this approach is different from a quantisation of the event horizon. In the present model, all horizon configurations can still be realized.

Generally, black holes emit particles via the Hawking mechanism [64,65]. The temperature of the radiation is:

$$T = \frac{\kappa}{2\pi} \quad , \quad (8)$$

with κ given by (5). From dimensional aspects we expect the entropy to be $S \propto \mathcal{A} M_f^{d+2}$. From Thermodynamics we know that,

$$\frac{\partial S}{\partial M} = \frac{1}{T} \quad , \quad (9)$$

where M is interpreted as the conserved energy of the system.

Inserting Eqs. (2), (5) and (8) one obtains a mass dependence of $1/T$ with the exponent $(1/(d+1))$. Integration yields

$$S(M) = \frac{d+1}{d+2} \frac{M}{T} = 2\pi \frac{d+1}{d+2} (M_f R_S)^{d+2} \quad . \quad (10)$$

In the limit where the energies of the emitted particles are small compared to the mass of the black hole the grand-canonical ensemble can be used. The number density of particles with energy ω is then

$$n(\omega) = \frac{1}{\exp \frac{\omega}{T} - 1} \quad . \quad (11)$$

with which one derives the higher dimensional analogue of the Stefan-Boltzmann law [66,67]

$$\varepsilon = \frac{\Omega_{(d+3)}^2}{(2\pi)^{d+3}} \Gamma(d+4) \zeta(d+4) T^{d+4} . \quad (12)$$

The spectrum of the emitted radiation has a maximum at frequencies of the order of the temperature. Thus, when the mass of the black hole approaches the Planck scale, the energy of the emitted quanta can no longer be neglected. Since we are interested in the late evaporation stage, when the black hole is small and hot, an appropriate statistical description is given by the micro-canonical ensemble [67–69]. Here, the single-particle number density is given by

$$n(\omega) = \frac{\exp[S(M - \omega)]}{\exp[S(M)]} , \quad (13)$$

where S denotes the entropy of the black hole.

Let us shortly examine the limit of huge black hole masses in the micro-canonical approach. The limit of the grand-canonical number density is Wien's limit, i.e. $n(\omega) = \exp(-\omega/T)$. In the micro-canonical case

$$\ln n(\omega) \approx -2\pi(M_f R_S)^{d+2} \frac{\omega}{M} = -\frac{4\pi R_S}{d+1} \omega = -\frac{\omega}{T} \quad (14)$$

in leading order in ω/M . Thus, Wien's limit is recovered.

Next we investigate the multi-particle spectrum in the micro canonical description:

$$n(\omega) = \sum_{j=1}^{\lfloor \frac{M}{\omega} \rfloor} \frac{\exp[S(M - j\omega)]}{\exp[S(M)]} , \quad (15)$$

with $\lfloor x \rfloor$ being the smallest integer next to x . This cut-off assures that the total energy of the emitted quantum does not exceed the mass of the black hole. After substituting $x = M - j\omega$ one obtains for the total energy density which is radiated off by the black hole

$$\varepsilon = \frac{\Omega_{(d+3)}}{(2\pi)^{3+d}} e^{-S(M)} \sum_{j=1}^{\infty} \frac{1}{j^{d+4}} \times \int_0^M e^{S(x)} (M - x)^{3+d} dx . \quad (16)$$

The evaporation rate per degree of freedom is given by

$$\frac{dM}{dt} = \frac{\Omega_{(d+3)}^2}{(2\pi)^{d+3}} R_S^{2+d} \zeta(d+4) e^{-S(M)} \times \int_0^M e^{S(x)} (M - x)^{(3+d)} dx . \quad (17)$$

Fig. 1 (thin dotted lines) shows the evaporation rate (17) as a function of the initial mass M of the black hole. In the limit $M \rightarrow \infty$, the micro canonical evaporation

rate reproduces the Hawking evaporation in $(d+3)$ space-like dimensions:

$$\begin{aligned} \lim_{M \gg M_f} \frac{dM}{dt} &= \mathcal{A}_D \frac{\Omega_{(d+3)}}{(2\pi)^{d+3}} \int_0^\infty \frac{\omega^{3+d} d\omega}{\exp(\omega T^{-1}) - 1} \\ &= \frac{\Omega_{(d+3)}^2}{(2\pi)^{d+3}} \Gamma(d+4) \zeta(d+4) R_S^{2+d} T^{d+4} , \end{aligned}$$

which is, using (8),

$$\begin{aligned} \lim_{M \gg M_f} \frac{dM}{dt} &= \frac{\Omega_{(d+3)}^2}{(2\pi)^{2d+7}} \left(\frac{d+1}{2} \right)^{d+4} \Gamma(d+4) \frac{\zeta(d+4)}{R_S^2} \\ &\propto M_f^2 \left(\frac{M}{M_f} \right)^{-\frac{2}{d+1}} . \end{aligned}$$

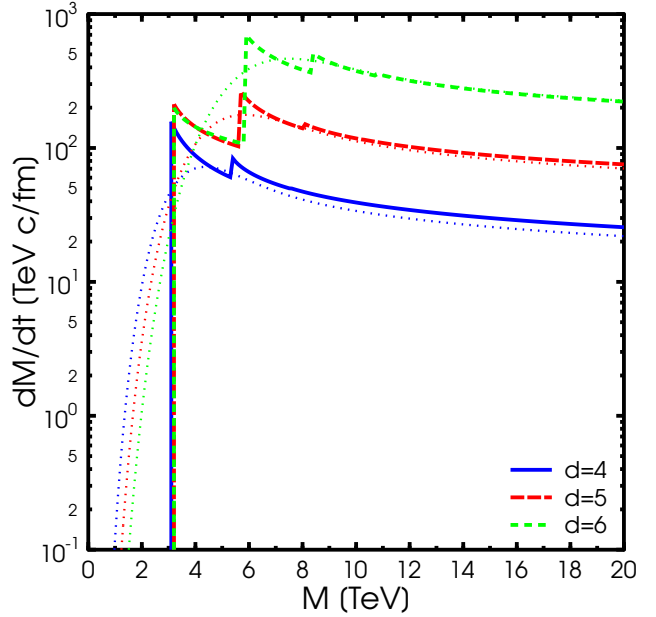


FIG. 1. Black hole evaporation rate per degree of freedom as a function of mass, from top to bottom for $d = 6, 5, 4$. The thin dotted lines show the standard micro-canonical scenario. The thick lines denote the calculations with the discrete energy spectrum.

Now we address the model with geometrical quantisation of the emitted radiation. The radiation of a black hole derived by semi-classical quantum field theory in curved space [70] yields a black body spectrum in $d+3$ dimensions. Spherical symmetry is taken into account by making the usual separation ansatz for the wave equation:

$$\psi(r, \Omega) = \frac{1}{r^{d+1}} Y_{lm}(\Omega) \phi(r) , \quad (18)$$

which factorizes the full wave function into an amplitude $1/r^{d+1}$, a radial wave function ϕ and a set of spherical harmonics Y_{lm} . Here Ω is an abbreviation for the

occurring $d + 2$ angles. The coefficients in the multipole expansion suppress the energy emission in higher order harmonics, therefore we consider only the $l = 0$ mode. In this case the dispersion relation is the usual one for massless particles $k^2 = \omega^2$. The boundary conditions of the black body lead to a restriction of the possible momenta¹.

This results in a geometrical quantisation of the momentum spectrum

$$k_l = \frac{\pi l}{R_S} \quad . \quad (19)$$

As a consequence, the multi-particle spectrum (15) is modified, since the emitted particles have energies in integer multiples of a minimal energy quantum $\Delta\omega = \pi/R_S$:

$$n(l) = \sum_{j=1}^{\lfloor \frac{M}{l\Delta\omega} \rfloor} \frac{\exp[S(M - jl\Delta\omega)]}{\exp[S(M)]} \Theta(M - l\Delta\omega) \quad . \quad (20)$$

Here the Θ -function cuts off the spectrum when the energy of one particle exceeds the mass of the black hole. The energy density of the radiation is derived by summation over momentum space

$$\varepsilon = \frac{\Omega_{(d+3)}}{(2\pi)^{3+d}} \Delta\omega \sum_{l=1}^{\lfloor \frac{M}{\Delta\omega} \rfloor} n(l) (l\Delta\omega)^{d+3} \quad . \quad (21)$$

In the limit of large black hole masses $M \gg M_f$ one has $\Delta\omega \rightarrow 0$ and regains the continuous emission spectrum from Eq. (21). The evaporation rate with respect to the geometrically quantised spectrum is shown in Fig. 1 (thick lines) and compared to the continuous spectrum case (thin dotted lines).

The spacing of energy levels gets smaller with increasing M , and whenever it is possible to occupy an additional level the evaporation rate exhibits a step. These steps naturally appear in spacings $\approx \pi$ TeV, because in the mass range of interest it is $R_S \approx 1/M_f$ – the exact value thereby depending on the number of extra dimensions.

The evaporation process occurs in quantised steps. It can not proceed further when the lowest lying quantum state allowed exceeds already the mass of the black hole. Evaporation is halted at a finite mass value for a certain fraction of initial masses above the fundamental scale. It should be noted that the transverse momentum spectra of the radiation is modified as compared with a simple extrapolation of the Hawking formula to small masses:

¹Note that the energy levels of the radiation in the monopole contribution are similar to the energy levels of a cubic black body radiator of side length $2R_S$ that was discussed in Ref. [71].

Here the final quanta possess energies of the order $1/R_S$, and not a continuous spectrum with $T(M \approx 1 \text{ TeV})$.

If the mass were slightly above the Planck scale, it would still be possible for the black hole to emit a particle carrying away most of its energy. This might leave a stable relic with mass below Planck mass. However, since physics below the Planck scale is unknown, we cannot be sure about the existence and properties of relics with masses below the fundamental scale. Thus, in the following we will focus on those relics with masses above the fundamental scale.

It is interesting to ask for the spectrum of final relic masses M_{relic} depending on the initial mass M_{initial} of the black hole. This relation is shown in Fig. 2 for the case $d = 3$. The most probable case is the exclusive emission of minimal energy quanta during the evaporation process, depicted by solid lines in Fig. 2. Black holes with $M_{\text{initial}} \leq 3M_f$ are stable with $M_{\text{relic}} = M_{\text{initial}}$. The inclusion of higher modes in the evaporation process, which becomes more important with increasing initial mass, is shown by the dashed and dotted lines. When going even further, to masses $M_{\text{initial}} \gg M_f$, the whole range of end masses would become accessible. Because the black holes accessible at the LHC would have mainly masses slightly above M_f , most of these black holes were stable with masses around M_f .

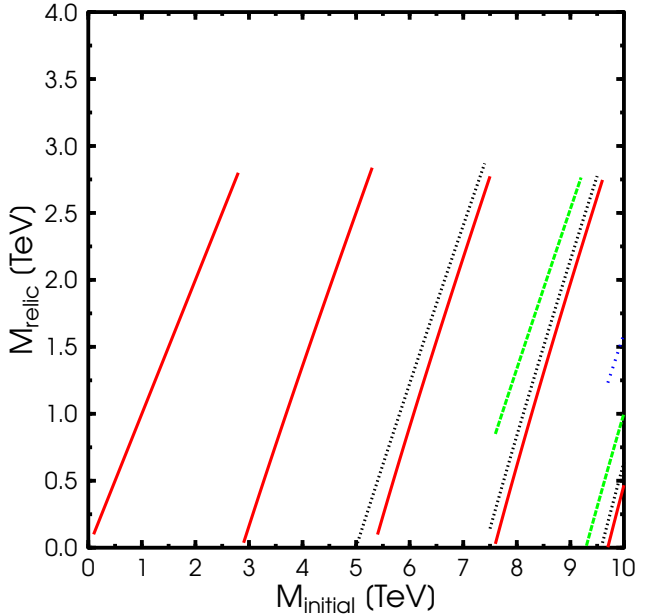


FIG. 2. Possible final relic masses (M_{relic}) after the evaporation process from a black hole of initial mass M_{initial} . The calculation is for $d = 3$ extra dimensions and $M_f = 1 \text{ TeV}$. The solid lines belong to the most probable case, other lines include the emission of higher modes.

Indirect constraints on the existence of black hole relics

can be obtained from the decay of primordial black holes. Their modified and dimension dependent energy spectrum influences observables, e.g. the cosmic microwave background and the baryogenesis [72,73]. Furthermore, the black hole relics from primordial density fluctuations may be a candidate for dark matter [74,75]. Upper limits on the relative contribution of those relics to the critical energy density in the universe are on the order of $\Omega_{\text{relics}} = 0.1 - 1$ [76]. Thus, the observation of relics in a collider experiment is of highest interest.

The final and most interesting question is: How could one observe these relics?

- A black hole relic with a mass of ≈ 3 TeV would have a spectrum that just fails to allow for a last emission of a quantum (cf. Fig. 2). Therefore, if its mass were increased only slightly by the energy ΔE , it would enable the black hole to evaporate again, emitting a high energetic quantum and leaving a tiny mass relic. This might result in a delayed flash of hard photons, leptons or QCD jets compared to the collision dynamics encountered at the LHC. The fraction of black holes evaporating in this manner can be estimated from the mass spectrum of black holes produced at the LHC and is $\Delta E/100 \text{ TeV}^{-1} \sim 10^{-4} - 10^{-5}$ for $\Delta E = 1 \text{ GeV}$. Note that relics from primordial black holes might also lead to observable air showers, if a black hole relic evaporates in the atmosphere.
- A certain fraction of the black holes produced in parton-parton collisions would carry a small charge of order e . This might allow to identify the charged black hole relics, e.g. by ionisation in a time projection chamber.
- The thermal evaporation spectrum would be much softer as expected in the literature [22] which assumes total decay of TeV black holes. However, the final stages would be governed by non-thermal particle emission.

To summarise, we have analysed the late stages of black hole evaporation, including the geometrical quantisation of the emitted radiation. In this model setting, the production of stable black hole relics would be possible at the LHC. These relics may be observable in a late burst of jets if they capture additional energy. Charged black hole relics may eventually be directly detected, e.g. in a TPC by ionisation.

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